$f = V \circ U$

$$\begin{cases} \lim\limits_{x\to 1} U(x) = \lim\limits_{x\to 1}(x-1)^2 = 0 \\ \lim\limits_{x\to 0} V(x) = \lim\limits_{x\to 0} \dfrac{\sin(\pi x)}{x} = \pi \end{cases}$$

$d'où \ \lim\limits_{x\to 1} f(x) = \pi$

$$g(x) = \begin{cases} f(x) & \text{si } x \neq 1 \\ \pi & \text{si } x = 1 \end{cases}$$

EX 7 :

$\Delta : y = -x + 2$

1)* $\lim\limits_{x\to -\infty} f(x) = 0$

* $\lim\limits_{x\to 0} f(x) = +\infty$

* $\lim\limits_{x\to 0^+} f(x) = -\infty$

* $\lim\limits_{x\to 2^-} f(x) = -\infty$

* $\lim\limits_{x\to 2^+} f(x) = +\infty$

* $\lim\limits_{x\to +\infty} f(x) = -\infty$

* $\lim\limits_{x\to +\infty} \dfrac{f(x)}{x} = -1$

* $\lim\limits_{x\to +\infty} (f(x) + x - 2) = 0$

2)

x	$-\infty$		0		1		2		$+\infty$
f'(x)		+	‖	+	0	-	‖	-	
f(x)		$+\infty$ ↗	‖	↗	1	↘	‖	↘	
		0	‖	$-\infty$		$-\infty$	‖	$-\infty$	

3)* $m \in]-\infty, 0] \cup \{1\}$

l'équation : f(x)=m admet trois solutions

* $m \in]0, 1[$

l'équation f(x)=m admet quatre solutions

* $m \in]1, +\infty[$

l'équation f(x)=m admet deux solutions

EX 8 :

1) On pose $U(x) = \dfrac{\pi x + 1}{x}$ et $V(x) = \cos(x)$

f=VoU

$$\left.\begin{array}{l} \lim\limits_{x\to -\infty} U(x) = \pi \\ \lim\limits_{x\to \pi} V(x) = -1 \end{array}\right\rangle \lim\limits_{x\to -\infty} f(x) = -1$$

2) $$\left.\begin{array}{l} \lim\limits_{x\to +\infty} \dfrac{3}{\sqrt{x}} = 0 \\ \lim\limits_{x\to 0} \sin(x) = 0 \end{array}\right\rangle \lim\limits_{x\to +\infty} f(x) = 0$$

3) $f(x) = \dfrac{\sin(\sqrt{x})}{x}$ *on pose* $X = \sqrt{x}$

$$\lim_{x\to 0^+} f(x) = \lim_{X\to 0^+} \frac{\sin(X)}{X^2}$$

$$= \lim_{X\to 0^+} \frac{\sin(X)}{X} \cdot \frac{1}{X} = +\infty$$

4) $f(x) = \dfrac{\cos(x^2-1) - 1}{(x^2-1)^2}$

$f = VoU$ *avec* $U(x) = x^2 - 1$ *et* $V(x) = \dfrac{\cos(x) - 1}{x^2}$

$$\left.\begin{array}{l} \lim\limits_{x\to 1} U(x) = 0 \\ \lim\limits_{x\to 0} V(x) = -\dfrac{1}{2} \end{array}\right\rangle \Rightarrow \lim_{x\to 1} f(x) = -\frac{1}{2}$$

EX 9 :

$x \in [0, +\infty[$

$f(x) = (\sqrt{x+2} - \sqrt{x}).\sin x$

1) $\sqrt{x+2} - \sqrt{x} = \dfrac{(x+2) - x}{\sqrt{x+2} + \sqrt{x}} = \dfrac{2}{\sqrt{x+2} + \sqrt{x}}$

d'où $f(x) = \dfrac{2.\sin x}{\sqrt{x+2} + \sqrt{x}}$

2) x>0

$$|f(x)| = \frac{2.|\sin x|}{\sqrt{x+2} + \sqrt{x}}$$

$$\sqrt{x+2} + \sqrt{x} \geq \sqrt{x} \Rightarrow \frac{1}{\sqrt{x+2} + \sqrt{x}} \leq \frac{1}{\sqrt{x}}$$

$$|\sin x| \leq 1 \Rightarrow 2|\sin x| \leq 2$$

d'où $\dfrac{2.|\sin x|}{\sqrt{x+2} + \sqrt{x}} \leq \dfrac{2}{\sqrt{x}}$

Qarray.tn -Aqra Blech- Qarray.tn